Abstract: The optical tomography problem presents some interesting difficulties for both experimental and theoretical work. This paper has attempted an overview of the theoretical problems for image reconstruction. In this paper we review some general approaches to inverse problems to set the context for optical tomography. An essential requirement is to treat the problem in a nonlinear fashion, by using an iterative method. The inverse problem is approached by numerical solutions methods using MathCad program. The Radon transform is the basic tool of the computerized tomography. In the sequel we introduce this transform, review some properties and present a numerical program for its inversion. We show some results that represent the most complex and realistic simulations of optical tomography yet developed.

1. Introduction

The development of an optical tomography imaging system would provide a functional image of local oxygenation.

This quest for a tomography modality follows the course of similar developments that saw the progression of X-ray radiography to X-ray computed tomography (CT), nuclear magnetic resonance spectroscopy (NMRS) to magnetic resonance imaging (MRI), and gamma cameras to single photon emission computed tomography (SPECT) and positron emission tomography (PET). More realistically, optical tomography follows the progression from the impedance camera to electrical impedance tomography (EIT). Due to the very time-consuming nature of data acquisition, considerable effort has been put into theoretical studies of the potential of optical tomography, going so far as to produce simulated images prior to the development of practical data acquisition systems, an order of development possibly unique in the development of medical imaging.

The optical tomography problem presents some interesting difficulties for both experimental and theoretical work. This paper has attempted an overview of the theoretical problems for image reconstruction.

We show some results that represent the most complex and realistic simulations of optical tomography yet developed. We suggest, in particular, that both time-resolved and intensity-modulated systems can reconstruct variations in both optical absorption and scattering, but that un-modulated, non-time-resolved systems are prone to severe artifact. We believe that optical tomography reconstruction methods can now be reliably applied to a wide variety of real clinical data. The expected resolution of the method is poor, meaning that it is unlikely that the type of high-resolution images seen in computed tomography or medical resonance imaging can ever be obtained. Nevertheless we strongly expect the functional nature of these images to have a high degree of clinical significance.
2. Radon transform and its numerical inversion

Definition of the Radon transform: we define the space of rapidly decreasing functions:
\[
S(R^n) = \left\{ f : R^n \to C \mid \forall p \in N, \forall \alpha \ multi-index, \sup_{x \in R^n} \| x^P \|^{\alpha} f \| < \infty \right\} \tag{1}
\]

The Radon transform for a function \( f \) in \( s(R^n) \) is defined by
\[
Rf(\xi) = \int_{s^n} f(x) dm(x)
\]
(2)

In this case \( \xi \) is a hyperplane and \( dm(x) \) is the Euclidian measure on it. The Radon transform of \( f \) is therefore a function defined on the multitude of hyperplanes in \( R^n \) noted by \( \Pi_n \).

Using \([5]\) we are reviewing a series of properties of the Radon transform.

**Hyperplane:** A hyperplane is characterized by a point \( \omega \in S^{n-1} \) and a number \( p \in R \) with the help of the equation \( \omega x = p \). The Radon transform \( Rf \) of function \( f \) therefore emerges as a function
\[
Rf : S^{n-1} \times R \to C
\]
given by the relation
\[
Rf(\omega, p) = \int_{\omega x = p} f(x) dm(x)
\]
(4)

having the property
\[
Rf(\omega, t) = Rf(-\omega, -p)
\]
(5)

Inverted mediation: for a function defined on \( \Pi_n \) we define \( I_{\phi} = \phi : R^n \to C \) through the relation
\[
I_{\phi}(x) = \int_{x \in \xi} \phi(\xi) d\xi = \frac{1}{\Omega_n} \int_{S^{n-1}} \phi(\omega, p) d\omega = \int_{O(n)} \phi(x + k\xi_0) dk
\]
(6)

where
\[
d\xi
\]
is a measure on the multitude of hyperplanes which pass through a point; it is invariant to rotations and thus normalized for a total measurement result of 1.

\[
\Omega_n = \frac{2\pi^{\frac{n}{2}}}{\Gamma \left( \frac{n}{2} \right)}
\]
is the area of sphere \( S^{n-1} \) in \( R^n \). \( dk \) is a Haar measure normalized on rotation group \( O(n) \). Transformation \( I \) shall be called inverted mediation.

**Inversion formulas**

\[
f(x) = \frac{\Gamma(1/2)}{(4\pi)^{n/2}} \left( -L \right)^{n-1/2} I(Rf)(x)
\]
(7)

\[
= \frac{\Gamma(1/2)}{(4\pi)^{n-1/2}} \left( -L \right)^{n-1/2} \int_{O(n)} Rf(x + k\xi_0) dk
\]

\[
= \frac{1}{\Omega_n} \frac{\Gamma(1/2)}{(4\pi)^{n-1/2}} \left( -L \right)^{n-1/2} \int_{S^{n-1}} Rf(\omega \cdot x) d\omega
\]
Take H as the Hilbert transform $Hg(p) = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{g(t)}{t-p} \, dp$, the integral being understood as the main Cauchy value for $g \in S(R^n)$ functions. H may be defined with the help of the Fourier transform by $F(Hg)(s) = \text{sgn}(s) \cdot F(g)(s)$.

We define operation $\Lambda$ for functions from $S_H(P_n)$ by

$$\Lambda \varphi(\omega, p) = \begin{cases} \frac{\partial^{n-1}}{\partial p^{n-1}} \varphi(\omega, p) & \text{If n is an odd number} \\ H p \frac{\partial^{n-1}}{\partial p^{n-1}} \varphi(\omega, p) & \text{If n is an even number} \end{cases}$$

We have

$$f(x) = \frac{1}{\Omega_n} \int_{R^n} \left[ \int_0^{2\pi} \int_{-\infty}^{\infty} e^{-isp} Rf(\omega, p)dp \right] e^{ix\omega} s^{n-1} ds \, d\omega$$

The inversion formula of the Fourier transform gives

$$f(x) = \frac{1}{(2\pi)^n} \int_{S^{n-1}} \left[ \int_0^{2\pi} \int_{-\infty}^{\infty} e^{-isp} Rf(\omega, p)dp \right] e^{ix\omega} s^{n-1} ds \, d\omega$$

where from the preceding formula can be recognized.

A.S. Fokas and R.G. Novikov proposed an inversion formula for the plane.

In point $(x_1, x_2)$ from the plane we have

$$f(x_1, x_2) = -\frac{1}{4\pi^2} \int_0^{2\pi} h_\rho(\rho, \theta) d\theta$$

where $h_\rho(\rho, \theta) = \frac{\partial}{\partial \rho} \int_{-\infty}^{\infty} Rf(\rho', \theta) d\rho'$ and $\rho = x_2 \cos(\theta) - x_1 \sin(\theta)$.

**Numerical algorithm**

For the reconstruction of function $f(x_1, x_2)$ in a given point $(x_1, x_2)$

1. Take all discreet data of Radon transform $Rf(\theta, \rho)$ for $\theta = \theta_j = \frac{2\pi}{N}, j=0,1,..N-1, \rho = \rho_i, i=0,1…N-1$ in the matrix $f_{i,j} = Rf(\theta_j, \rho_i)$.
2. Interpolate function $Rf(\theta, \rho)$ either linearly or by spline functions (we used a linear interpolation)
3. Compute $\int_{-\infty}^{\infty} \frac{Rf(\rho', \theta)}{\rho' - \rho} d\rho'$ for $\theta = \theta_j$ using the interpolation at point 2.
4. The derivative result from 3 gives

$$h_j(\rho) = \sum_i \frac{f_{i+1,j} - f_{i,j}}{\rho_{i+1} - \rho_i} \ln \left| \frac{\rho - \rho_{i+1}}{\rho - \rho_i} \right| + \frac{f_{i+1,j} - f_{i,j}}{\rho - \rho_{i+1}}$$

5. For $\rho = x_2 \cos(\theta_j) - x_1 \sin(\theta_j)$ in $h_j(\rho)$ the result is $h_j$.
6. Compute $f(x_1, x_2) = -\frac{1}{4\pi^2} \int_0^{2\pi} h_\rho(\rho, \theta) d\theta$ by approximation from $S = -\frac{1}{4\pi^2} \frac{2\pi}{N} \sum_j h_j$.

Generally speaking, function reconstruction is made in a matrix of points, where each value in a given point represents the grey intensity in the respective point. The program used Mathcad, by loading an image, then the matrix was read with the grey intensities for each point, and the
density \( f(x_1, x_2) \) resulted from there by interpolation. The function was interpolated with the option that the image is enclosed in the disc with a ray 1. The Radon transform \( Rf(\theta, \rho) \) resulted from the integration on a number of directions and, for each direction, for a number of equidistant parallels, thereby giving the Radon data matrix \( \{f_{i,j}\} \). Image reconstruction is performed according to the preceding algorithm, where values smaller than an \( \varepsilon \) of differences \( \rho - \rho_i \) are replaced with \( \varepsilon \).

3. Results and discussions

The techniques of non-invasive optical coherent tomography and its novel branch – polarization-sensitive, based on measuring 2D depth-resolved Mueller matrix elements with high space resolution and high scanning speed represent a very important direction of such diagnostics.

Measuring the Stokes vector of light, scattered by biological tissues, and calculating the corresponding Mueller matrix enables to obtain the most complete (statistically averaged at the whole totality of bio-object’s in homogeneities) information on their polarization properties. The analysis of such information is promising for the studying the macrostructure of various BT within the tasks of biomedical diagnostics of their physiological state.

The Mueller-matrix images analysis enables to obtain the important biomedical information about not only biological tissues geometrical macrostructure, but also about the value and coordinate distributions of optical parameters of their architectonic nets anisotropy, made up by geometrically self-similar bundles of collagen myosin, etc.

The optical circuit of measuring the biological tissue polarized structure is given on the fig. 1. The main optical elements of the polarimeter are the He-Ne laser (1) \((\lambda = 0.6328 \mu, 10mW)\), the Polaroid \( P (4) \), two quartz-made quarter-wave \((\frac{\lambda}{4})\) plates \((3, 5, 8)\) \((4^{th}\) order) and the Polaroid \( A (9) \) as the analyser. By setting the polarisation plane of \( P \) parallel to the optical axis of the first quarter-wave plate the object under test (a tissue sample (6)) is illuminated by a linearly polarised light beam. With the analyser similarly coupled with the second quarter-wave plate.

The polarization illuminator consisted of a quarter-waved plates 3; 5 and the polarizer 4 forms the illuminating beam with an arbitrary azimuth of \( 0 \leq \alpha_0 \leq \pi \) or with the polarization ellipticity of \( 0 \leq \beta_0 \leq 0.5\pi \).

The polarization images of the biological tissue done with the help of micro objective 7 have been projected into the plane of a light-sensitive area \((800x600)\) of CCD-camera 10, which provided the measuring range of structural elements of the biological tissue (6) for the following scales \(2 \ \mu m - 2000 \ \mu m \).

The analysis of the biological tissue images has been done by the polarizer 9.

The methods of determining the coordinate distribution of the polarization singularities of the biological tissue image consists in the following sequence of action: the minimum and maximum levels of the image intensity for each separate pixel of CCD-camera and the corresponding to them rotation angles are determined by the rotating an axis of the analyzer transmission 8 in the limits of \( \Theta = 0 - \pi \).
Figure 1. Optical scheme of experimental polarimetry: 1-laser source; 2-afocal system; 3,5,8-quarter-waved (λ/4) plates; 4.9-linear polarizer; 6–biological tissue; 7-micro objective; 10-CCD camera.

Figure 2 demonstrates the series of experimentally determined topological distribution and showing the real images captured with CCD camera and the reconstructed images with inverse Radon transform.

Experimental measuring of biological tissue polarization-correlation maps provides the possibility to do the analysis of the influences of orientation architectonics peculiarities of physiologically different biological tissue in a polarization map speckle-image structure. Exist an interconnection between the coordinate structure pathological changes diagnostics using complex degree of mutual polarization of biological tissue images and the optical-geometrical parameters of their physiologically normal and pathologically changed birefringent architectonics nets.
4. Conclusions

On the basis of polarization measurements technique of 2-dimensional distributions of the coordinate structure pathological changes diagnostics using complex degree of mutual polarization of biological tissue images the interconnection between the statistical moments of the coordinate structure pathological changes diagnostics using complex degree of mutual polarization values and the optical-geometrical structure of degenerative and/or dystrophically changed the architectonics nets of biological tissue. The obtained information concerning the polarization-correlation structure of images correspond to different biological tissue morphological structure can be used for elaboration of new methods for their analysis of physiological state. The modified biochemical parameters are correlated with the cell membrane modifications detectable even by optical methods.

By comparing the real and reconstruction images we can conclude that the algorithm is not good enough and that it must be revised. In [3] the authors used a similar algorithm, but they used an interpolation with spline functions. According to examples included therein, reconstruction is much more precise, whereas the computation takes longer time. In a future study we intend to compare various inversion methods.

Bibliografie